The angle of rotation is defined as the angle between the normal vector of the plane defined by the 3 LVDT deflections and a vertical vector.

Plane equation:

\[ Ax + By + Cz + D = 0 \]  \hspace{1cm} (1)

Angle between the normals of two planes\(^1\):

\[ \cos \tau = \frac{A_1 A_2 + B_1 B_2 + C_1 C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \sqrt{A_2^2 + B_2^2 + C_2^2}} \]  \hspace{1cm} (2)

Plane passing through Three Points \( \mathbf{P}_i, \mathbf{P}_j, \mathbf{P}_k \) (Tuma, p. 50)

\[
\begin{bmatrix}
  y_i & z_i & 1 \\
  y_j & z_j & 1 \\
  y_k & z_k & 1 \\
\end{bmatrix}
\begin{bmatrix}
  x_i & y_i & 1 \\
  x_j & y_j & 1 \\
  x_k & y_k & 1 \\
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
\end{bmatrix}
= \begin{bmatrix}
  1 \\
  1 \\
  1 \\
\end{bmatrix}
\]

Horizontal Plane Equation: \( z = 0 \)

**LVDT readings**

Three points, 120° apart when projected in the X,Y plane:

\[
\mathbf{P}_1 = (R,0,\delta_i) \]  \hspace{1cm} (4)

\[
\mathbf{P}_2 = \left( \frac{-R}{2}, \frac{R \sqrt{3}}{2}, \delta_2 \right) \]  \hspace{1cm} (5)

\[
\mathbf{P}_3 = \left( \frac{-R}{2}, \frac{-R \sqrt{3}}{2}, \delta_3 \right) \]  \hspace{1cm} (6)

Where

- \( R \) = radius of specimen
- \( \delta_i \) = peak deflection of LVDT,

---

Equation for LVDT Plane

Solving determinants in Equation 3 using coordinates from Equations 4, 5, and 6:

\[
A = \sqrt{3} R \left( \frac{\delta_2}{2} + \frac{\delta_3}{2} - \delta_1 \right) \tag{7}
\]

\[
B = \frac{3}{2} R (\delta_3 - \delta_2) \tag{8}
\]

\[
C = \frac{3\sqrt{3}}{2} R^2 \tag{9}
\]

\[
D = -\frac{\sqrt{3}}{2} R^2 (\delta_1 + \delta_2 + \delta_3) \tag{10}
\]

Angle of Rotation, \( \theta \) (Angle between normal of LVDT plane and vertical)

Substituting Equations 7, 8, 9, and 10 into Equation 2:

\[
\cos \theta = \frac{\frac{3}{2} R}{\sqrt{\delta_1^2 + \delta_2^2 + \delta_3^2 - \delta_1 \delta_2 - \delta_1 \delta_3 - \delta_2 \delta_3 + \frac{9}{4} R^2}} \tag{11}
\]

In terms of diameter, \( D \):

\[
\cos \theta = \frac{\frac{3}{4} D}{\sqrt{\delta_1^2 + \delta_2^2 + \delta_3^2 - \delta_1 \delta_2 - \delta_1 \delta_3 - \delta_2 \delta_3 + \frac{9}{16} D^2}} \tag{12}
\]

Axis of Rotation

The axis of rotation is the intersection of the LVDT plane with the horizontal plane:

\[
z = \frac{\delta_1 + \delta_2 + \delta_3}{3} \tag{13}
\]

The equation for the intersection of two planes in the X,Y plane is (Tuma, p. 51):

\[
\begin{vmatrix} C_1 & C_2 \\ A_1 & A_2 \end{vmatrix} x + \begin{vmatrix} C_1 & C_2 \\ B_1 & B_2 \end{vmatrix} y + \begin{vmatrix} C_1 & C_2 \\ D_1 & D_2 \end{vmatrix} = 0 \tag{14}
\]

Substituting Equations 7, 8, 9, 10, and 13 into Equation 14 results in the equation for the axis of rotation:

\[
\sqrt{3} R \left( \frac{\delta_2}{2} + \frac{\delta_3}{2} - \delta_1 \right) x + \frac{3}{2} R (\delta_3 - \delta_2) y = 0 \tag{15}
\]