Using Remote Sensor Data to Estimate Pavement Performance Models

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ABSTRACT
We propose state-space specifications of autoregressive moving average models and structural time series models as a framework to develop and estimate incremental performance/deterioration models for transportation infrastructure facilities. State-space specifications are consistent with the infrastructure management using latent performance approach, which means that they rigorously account for uncertainty in forecasting infrastructure condition when data are gathered using multiple technologies. Moreover, these specifications fit the optimization framework using time series analysis to support maintenance and rehabilitation decision-making, which means that they constitute an alternative to the use of Markovian transition probabilities. Through an empirical study, we verify that the proposed methodology can be used to generate infrastructure condition forecasts when data are gathered with multiple technologies. The models in the study are estimated using deflection and pressure measurements generated by sensors embedded in an asphalt pavement. Analysis of the results corroborates earlier findings that question the universal validity of the Markovian assumption in the context of infrastructure deterioration.
INTRODUCTION
Optimization models to support maintenance and rehabilitation (M&R) investment decisions for transportation infrastructure must evaluate both the short and long-term economic consequences associated with these investments. This involves processing data related to current infrastructure condition and using them to forecast the effect of M&R decisions on future condition. The economic consequences associated with these decisions are then predicted by assuming a correspondence between infrastructure condition and costs. Information about current infrastructure condition is obtained by collecting distress measurements. Examples of distresses on transportation infrastructure include roughness, type and extent of cracking, rut depth and profile, extent of surface patching, on pavements; and cracking, spalling, and chloride contamination on bridge decks. Information about future condition, i.e., condition forecasts, is generated with performance models. A performance model is a statistical expression that relates condition to a set of explanatory variables such as design characteristics, traffic loading, environmental factors, and history of M&R investments.

From the previous paragraph, it follows that the ability to generate accurate condition forecasts is an essential part of developing efficient M&R policies. Ben-Akiva and Ramaswamy (1) introduced the latent performance modeling approach to rigorously address the problem of forecasting condition when multiple technologies are used to collect condition data. The key feature of the approach is that a facility's condition is represented by latent/unobservable variables that capture the ambiguity that exists in defining, and consequently in measuring infrastructure condition. Distress measurements are related to the latent condition through a measurement error model that accounts for systematic and random errors in the data-collection process, as well as, for the relationships between different technologies and distress measurements. Latent performance models also include a structural model that describes the relationship between a set of explanatory variables and infrastructure condition. Empirical studies by Ben-Akiva and Ramaswamy (1) and by Ben-Akiva and Gopinath (2) have shown that latent performance models are appropriate to generate condition forecasts of transportation infrastructure, i.e., the goodness-of-fit measures are better than those reported using other statistical methods. This, in turn, lead to Madanat and Ben-Akiva (3) that included latent performance models in a framework to support M&R decision-making by formulating the underlying optimization problem as a latent Markov Decision Process (MDP).

While providing a rigorous framework to account for uncertainty (in the deterioration and in the data-collection process), the latent MDP suffers from computational limitations that make it impractical to support M&R decision-making for transportation infrastructure when multiple technologies are used simultaneously to measure (different) distresses. As discussed in Durango-Cohen (4), these limitations are derived from the fact that the state and decision variables in the model are defined over discrete sets. More importantly, these limitations are of practical significance as a plethora of advanced inspection technologies, e.g., sensors, satellite imaging, video, laser, and radar, have become commonplace in evaluating and measuring distresses on transportation infrastructure.

To address the computational shortcomings of the latent MDP, Durango-Cohen and Tadepalli (5) proposed a discrete time, stochastic optimal control framework to support M&R decision-making when condition data are gathered using multiple (advanced) inspection technologies. The framework consists of two components: a state-estimation problem that involves processing arrays of condition data and using them to develop condition forecasts; and an optimization problem whose solution yields M&R policies. The variables in the framework
are defined over continuous spaces and the deterioration process is modeled as a time series. Time Series Analysis provides a broad class of incremental performance models that can be used to represent the evolution of infrastructure performance, and that can be included in the aforementioned framework to support M&R decision-making.

This paper complements the work by Durango-Cohen and Tadepalli (5). Specifically, we describe and compare two classes of time series models to represent the evolution infrastructure performance: AutoRegressive Moving Average (ARMA) models and structural time series models. We consider state-space specifications of the models because they are consistent with the latent performance modeling approach by Ben-Akiva and Ramaswamy (1), and because they fit the optimization framework by Durango-Cohen and Tadepalli (5). Through an empirical study, we verify that the proposed methodology is valid to forecast infrastructure performance when condition data are gathered with multiple technologies. The models in the study were estimated using deflection and pressure measurements generated by sensors embedded in an asphalt pavement section. The data was collected and generously provided by MnRoad, the road research division of the Minnesota Department of Transportation. The estimation results show that the history of the deterioration process is statistically significant, which means that the Markovian assumption that underlies the estimation of transition probabilities for the (latent) MDP framework may not be appropriate.

The remainder of the paper is organized as follows. LITERATURE REVIEW provides an overview of the MDP approach and discusses characteristics that make this approach statistically unattractive to forecast the performance of transportation infrastructure. We also review state-space specifications of time series models. METHODOLOGY describes the methodology used in this paper which consists of the formulation and estimation of the infrastructure performance models as ARMA or structural time series models. EMPIRICAL STUDY details the empirical study used to validate the methodology. A summary of the contributions of the paper is provided in CONCLUSIONS.

LITERATURE REVIEW

Optimization models to support M&R decision-making for transportation infrastructure rely on incremental performance models to forecast the effect of these decisions on changes in condition. These changes are added to previous condition in order to estimate new condition, i.e., the set of explanatory variables in incremental performance models includes lagged, dependent variables. As stated in the previous section, state-of-the-art optimization models to support M&R investment decisions are formulated as a (latent) MDP. We begin this section by reviewing the performance models embedded in (latent) MDP formulations. We then discuss characteristics that make these models statistically unattractive to forecast the performance of transportation infrastructure and introduce state-space specifications of time series models as an alternative.

Markovian Transition Probabilities

The (latent) MDP framework assumes that condition is represented with a discrete variable and that the deterioration process is represented with transition probabilities of the form:

\[
\text{Prob}(X_{t+1} = j | X_t = i, A_t = a), \quad \forall i, j \in S, a \in \mathcal{A}
\]

where:

- \(X_t\) is the random variable (defined over the discrete set \(S\)) used to represent a facility's condition at the start of period \(t\); and
A_t is the decision variable (defined over the discrete set \( A \)) used to represent the M&R decision selected for period t.

The transition probabilities are said to be Markovian because they do not depend on the history of the process, i.e., \( X_t \) is the only lagged, dependent variable used to forecast the condition at \( t+1, X_{t+1} \). The MDP framework has broad appeal because it provides a rigorous approach to account for the uncertainty inherent in forecasting infrastructure condition, and because optimal policies can be obtained by solving a linear program (c.f. (6), (7), (8)). Unfortunately, the efficacy of transition probabilities as an approach to forecast infrastructure condition is suspect. Among other reasons, this is due to the fact that the framework requires a discrete, single-dimensional representation of a facility's condition. These characteristics make the framework statistically unattractive to forecast the performance of transportation infrastructure because:

- The quality of condition forecasts depends critically on the scheme used to discretize the state (and the decision) variable(s). Latent MDP formulations also require a discrete representation of the distress measurements. The problem is that any scheme used to discretize these elements involves arbitrarily partitioning their range into a set of mutually exclusive and collectively exhaustive intervals. The partition, i.e., the number and length of the intervals, in turn, determine the precision and accuracy of the condition forecasts.
- As explained in Durango-Cohen (4), the framework does not provide the flexibility to increase the number of explanatory variables in the performance models, i.e., \( X_{t+1} \) is fully determined by \( X_t \) and \( A_t \). This means that it is not possible to represent a facility's state as a multi-dimensional vector. This is because the number of parameters/probabilities that need to be estimated increases exponentially with the number of explanatory variables. Relaxing the Markovian assumption by including multiple lagged, dependent variables in the state representation is an example of why this flexibility may be desirable. This issue is of practical significance as empirical studies in pavement management (c.f. (1)), and in bridge management (c.f. (9), (10)) have shown that the physical deterioration of transportation infrastructure is not Markovian. The empirical results presented in Empirical Study of this paper also show that the dependence of distress measurements (deflection and pressure) on the history of the deterioration process is statistically significant. The implication is that condition and cost forecasts generated with Markovian models may be error-prone and M&R policies obtained with these models may be inappropriate.
- The framework does not allow for the inclusion of independent, exogenous variables in the statistical model. These variables may be used to capture the effect of environmental factors, design characteristics, traffic loading, etc. on the deterioration of transportation infrastructure.

The lack of “statistical sophistication” associated with Markovian transition probabilities is well recognized to the point that it has spawned two distinct research areas: one involving the development and estimation of models for performance prediction purposes (c.f. (1), (2)); and one involving the development and estimation of transition probabilities that can be embedded in optimization models to support M&R decision-making (c.f. (9), (10)). Here, we emphasize that many the approaches to estimate transition probabilities are very sophisticated. The problem is that, ultimately, they lead to a framework that is both statistically unattractive as described above, and computationally inefficient as described in Durango-Cohen (4). This is perhaps the most
important contribution of the optimization framework presented in Durango-Cohen and Tadepalli (5), where facility performance is modeled as a time series, because it provides an integrated and sophisticated framework to simultaneously address performance modeling and M&R decision-making.

State-Space Specifications for Time Series Models

Here, we describe our approach to model infrastructure performance as a time series. We adopt state-space specifications to estimate time series models. These specifications are consistent with the latent performance modeling approach by Ben-Akiva and Ramaswamy (1) which, as discussed in INTRODUCTION, has been shown empirically to be adequate to represent the evolution of transportation infrastructure. State-space models, originally postulated for the optimal control of engineering systems, have been adapted to analyze time series. The general form of a state-space model is:

\begin{align}
X_{t+1} &= g_t X_t + h_t A_t + \xi_{t+1} \\
Z_t &= H_t X_t + \zeta_t
\end{align}

where \(X_t\) is vector used to represent a system's state, e.g., a facility's state. As in Ben-Akiva and Gopinath (2) the vector might, for example, include a component to represent functional performance and a component to represent structural fitness. The additional components in the vector can also be used to include lagged, dependent variables to account for the effect of the history of the process on the evolution of the system, i.e., to relax the Markovian assumption. The components of a state vector varie with the specification of the models and will be described later in the paper. \(A_t\) is a vector of exogenous inputs that may include a maintenance/investment rate, environmental factors, or traffic loading. \(Z_t\) is a vector used to represent the distress measurements. In this paper, we assume \(K\) technologies are used to measure the facility condition and the measurement and the error from the \(k^{th}\) technology at time \(t\) are \(Z_t^{(k)}\) and \(\xi_t^{(k)}\) respectively. Finally, \(\epsilon_t\) and \(\xi_t\) represent random error terms that are assumed to follow Gaussian Distributions with finite second moments.

Equation (2) is the system equation (or state equation) and Equation (3) is the measurement error model (or observation equation). The vector \(\xi_t\) is of particular interest because it captures systematic and random errors in the data-collection/measurement process. Without loss of generality, we follow the assumption of Humplick (11) where the errors are attributed to the measurement technologies. Equation (3) could be univariate or multivariate depending on the number of distresses being measured. The relationships between the distress measurements are captured by the covariance matrix of \(\xi_t, \Sigma_{\xi_t}\).

Two classes of time series models are used in the present study: ARMA models and structural time series models. The classical approach to ARMA time series models are developed by Box and Jenkins. The Box-Jenkins approach is based on the assumption of stationary data. This assumption is hard to justify since no overwhelming evidence shows that data from social or engineering systems is stationary. For non-stationary data, differencing techniques must be applied to obtain stationary data. However, the elimination of trend or seasonality by differencing may be a drawback when trend or seasonality is of interest, as may be the case when modeling the performance of transportation infrastructure. This is an additional benefit of state-space specifications because they do not rely on stationary data. The estimation of ARMA models to represent the performance of transportation infrastructure is described in Formulation of ARMA(p, q) Models.
We also consider structural time series models as described in Harvey (12) and Durbin and Koopman (13). This class of time series model involves separating the data into meaningful components, i.e., trend, cycle, seasonality, and irregularity/random error (referred to as residuals in ARMA model literatures). One of the advantages of structural time series models is that they do not rely on the assumption of stationary data. Another advantage is that the components are easy to explain and interpret (12). Similar to ARMA models, structural time series models can be specified conveniently as state-space models. The estimation of structural time series models is described in **Formulation of Structural Time Series Models**.

**METHODOLOGY**

In this section, we provide a brief overview of the time series models that we propose to represent the performance of transportation infrastructure. We also explain the procedure that is used to estimate the parameters for the different models and the approach to select between different models. To simplify the notation and to be consistent with the data used in the empirical study presented in **EMPIRICAL STUDY**, we exclude exogenous variables from the presentation.

**Formulation of ARMA(p, q) Models**

An ARMA(p, q) model with a facility’s state and K measurements has the following form. Equation (4) governs the transition behavior of the system and Equation (5) captures the relationship between the facility’s state and its measurements.

\[
x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_p x_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots + \theta_q \epsilon_{t-q}, \quad t = 1, 2, \ldots, T
\]

\[
z_t^{(k)} = \lambda_k x_t + \xi_t^{(k)}, \quad k = 1, 2, \ldots, K, \quad t = 1, 2, \ldots, T
\]

We let \( d = \max \{p, q+1\} \), \( \phi_i = 0 \) if \( i > p \), and \( \theta_j = 0 \) if \( j > q \). The following equations can be used to represent an ARMA(p, q) model in state-space form.

\[
X_t = \begin{bmatrix}
  x_t \\
  x_t^{(d)} \\
  \vdots \\
  x_t^{(d-1)} \\
  x_t^{(d-1)(d)}
\end{bmatrix}
= \begin{bmatrix}
  \phi_1 & 0 & \cdots & 0 & 0 \\
  \phi_2 & 0 & \cdots & 0 & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  \phi_{d-1} & 0 & \cdots & 0 & 1 \\
  \phi_d & 0 & \cdots & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_t^{(d)} \\
  x_t^{(d-1)} \\
  \vdots \\
  x_t^{(2)} \\
  x_t
\end{bmatrix}
+ \begin{bmatrix}
  1 \\
  \theta_1 \\
  \vdots \\
  \theta_{d-2} \\
  \theta_d
\end{bmatrix}
\epsilon_t
\]

\[
Z_t = \begin{bmatrix}
z_t^{(1)} \\
  z_t^{(2)} \\
  \vdots \\
  z_t^{(K-1)} \\
  z_t^{(K)}
\end{bmatrix}
= \begin{bmatrix}
  \lambda_1 & 0 & \cdots & 0 & 0 \\
  \lambda_2 & 0 & \cdots & 0 & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  \lambda_{K-1} & 0 & \cdots & 0 & 0 \\
  \lambda_K & 0 & \cdots & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_t \\
  z_t^{(1)} \\
  \vdots \\
  z_t^{(K-1)} \\
  z_t^{(K)}
\end{bmatrix}
+ \begin{bmatrix}
  \xi_t^{(1)} \\
  \xi_t^{(2)} \\
  \vdots \\
  \xi_t^{(K-1)} \\
  \xi_t^{(K)}
\end{bmatrix}
\]

\[
z_t^{(1)} - | \lambda_1 0 0 0 \ldots 0 | x_t + \xi_t^{(1)}
\]

The parameters to be estimated are \( \{ \phi_1, \ldots, \phi_d, \theta_1, \ldots, \theta_d, \sigma_\epsilon^2, \lambda_1, \ldots, \lambda_K, \Sigma_\xi \} \). Equation (6) represents the system equation. The first element in the state vector, \( x_t \), represents the true facility condition at time \( t \). The equivalence of this equation and Equation (4) is easy to check. First, \( x_t^{(d)} \) of the bottom row in Equation (6) is shifted backward by one time step and then inserted into the second from bottom row. Second, the substituted \( x_t^{(d-1)} \) is shifted to time \( t-1 \) and inserted into the third from bottom row. Similarly, the process continues until \( x_t^{(2)} \) is time shifted and inserted into the first row and a equation that contains only \( x \) is obtained. As a result, this equation would be exactly the same as Equation (4). Equation (7) is the measurement error.
model for K distress measurements (Equation (8) is for the special case of a single distress measurement). The true condition is generally set to be equal to one of the distress measurements, the reference measurement. This involves setting $\lambda_i = 1$ in Equation (7) or (8). As stated earlier, $\varepsilon_t$ and $\xi_t$ are assumed to follow Gaussian Distributions with finite second moments. We also assume, without loss of generality, that they have zero means.

**Formulation of Structural Time Series Models**

Structural time series models consist of components that capture trend, seasonality, and deterministic or stochastic irregularity. Each component is explained below. Lagged, dependent variables can also be included to capture the dependence on the history of the process. A general representation of a structural time series model is shown below in Equation (9) and Equation (5). This representation also has one facility's state and K measurements. The special case with no lagged, dependent variables in the model is obtained by setting $\phi_1 = \phi_2 = \ldots = \phi_p = 0$ and $p=1$.

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_p x_{t-p} + \mu_t + \gamma_t + \varepsilon_t, \quad t = p+1, \ldots, T$$  \hfill (9)

In the above equations, $\mu_t$ is the signal level of the system at time $t$, $\beta_t$ is the trend at time $t$, and $\gamma_t$ is the seasonal component at time $t$. $\eta_t$, $\zeta_t$, and $\omega_t$ are random error terms that can be embedded in each of the structural components given in Equations (10), (11) and (12). When the variance in the random error terms is zero, then the components are deterministic; otherwise, they are stochastic. Equation (9) is also equal to Equation (13) below except the time subscripts and the use of it will be explained shortly. $\varepsilon_t$ is the random error of the transition of the whole system equation.

$$\text{Level: } \mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t$$ \hfill (10)

$$\text{Trend: } \beta_t = \beta_{t-1} + \zeta_t$$ \hfill (11)

$$\text{Seasonality: } \sum_{j=0}^{24} \gamma_{t-j} = \omega_t$$ \hfill (12)

$$\text{Lagged System Equation: } x_{t+1} = \phi_1 x_t + \ldots + \phi_p x_{t-p+1} + (\mu_t + \beta_t + \eta_{t+1}) + (\omega_{t+1} - \gamma_t - \gamma_{t-1} - \ldots - \gamma_{t-24}) + \varepsilon_{t+1}$$ \hfill (13)

An alternative approach to handle lagged dependent variables is to treat them as exogenous variables, for example, Madanat et al. (9). However, when we forecast more than one step in time series, all or some of the lagged dependent values must be replaced by predicted values. If we treat the lagged dependent variables as exogenous values, these values have no contribution to prediction mean square error (MSE) when we are looking for the best predictors. On the other hand, our specification incorporates the lagged dependent variables in the state vector and uses Equation (13) to predict automatically. The predicted values that used as lagged dependent variables have contribution when calculating prediction MSE. Thus, our approach is more statistically rigorous than treating the lagged dependent values as exogenous values.

Combining all the components above, the state vector and system equation of a state-space model for a structural time series are presented in Equations (14) and (15). In the equations, $\varepsilon_t$, $\eta_t$, $\zeta_t$, and $\omega_t$ are assumed independent Normally distributed random variables with zero mean and finite variance. If no lagged dependent variables are used, $\phi_1 = \phi_2 = \ldots = \phi_p = 0$ and $p=1$. The measurement error models are the same as ARMA models described in Equation (7). As with ARMA models, the latent condition is generally set to be equal to a reference measurement. The parameters to be estimated in this specification are $\{ \phi_1, \ldots, \phi_p, \sigma^2, \sigma^2, \sigma^2, \sigma^2, \lambda_1, \ldots, \lambda_K, \Sigma \}$. 

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Estimation and Model Selection

The objective of the estimation procedure for the models presented earlier is to minimize the prediction errors. The prediction error at time t, $e_t(\psi)$, is the difference between the distress measurement predicted using the parameter vector $\psi, Z_t(\psi)$, and the observed value, $Z_t$ as shown in Equation (16). If only one distress measurement is considered, the criterion corresponds to the sum of square errors Equation (17). If the number of the distress measurements is greater than one, the measure could be any quadratic function, $l(.)$, of the predicted errors over time as is shown in Equation (18). Equation (19) specifies that the estimated parameters are those that minimize the criterion for the given data. In this paper, we use MATLAB to estimate the models using a robust quadratic prediction error.

$$
\begin{align*}
X_t &= \\
&= \begin{bmatrix}
\phi_1 & \phi_2 & \ldots & \phi_p \\
1 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_t \\
x_{t-1} \\
\vdots \\
x_{t-p+1} \\
\end{bmatrix}
+ \begin{bmatrix}
\beta_{t0} \\
\beta_{t1} \\
\gamma_t \\
\gamma_{t-1} \\
\gamma_{t-2} \\
\vdots \\
\gamma_{t-p+2} \\
\end{bmatrix}
\begin{bmatrix}
\eta_{t+1} \\
\omega_{t+1} \\
\epsilon_{t+1} \\
\end{bmatrix}
\end{align*}
$$

(14)

$$
X_{t+1} = \\
= \begin{bmatrix}
0 & 1 & 1 & -1 & -1 & \ldots & -1 \\
0 & 1 & 0 & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 1 & 0 & 0 & 0 & \ldots & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_{t+1} \\
x_{t} \\
\vdots \\
x_{t-p+1} \\
\end{bmatrix}
+ \begin{bmatrix}
\eta_{t+1} \\
\omega_{t+1} \\
\epsilon_{t+1} \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
\gamma_t \\
\gamma_{t-1} \\
\gamma_{t-2} \\
\vdots \\
\gamma_{t-p+2} \\
\end{bmatrix}
\end{align*}
$$

(15)

Generally, $V_T$ decreases as the number of estimated parameters of $\psi_T$ in Equation (17) or (18) increases. However, $V_T$ will decrease even if the number of estimated parameters is greater than the number of the “preferred” model, which is referred to as “overfit” (14). Therefore, in practice the objective is to find the balance of model fit and the number of parameters. Two measures of goodness-of-fit are commonly used to compare the model specifications and choose between them. These are Final Prediction Error (FPE) and Akaike's Information Criterion (AIC), both proposed by Akaike. These criteria are modifications of Equation (17) or (18) that include penalties that increase with the number of model parameters. Using these two measures, the models with too many parameters are not chosen; hence FPE and AIC are used to select the desired model specifications in the empirical study.
We note that although autocorrelation and partial autocorrelation functions are commonly used for selecting ARMA models, they are not applicable in state-space form. Instead, backward elimination will be used and demonstrated in the next section. After a model is selected, we validate the model by examining the error between the one-step-ahead predicted values and the actual observed data. We also conduct the residual analysis to check the serial correlations of the residuals. The serial correlations of the residuals are expected to be insignificant.

EMPIRICAL STUDY

The purpose of this study is to describe the process of preparing the required data and illustrate the estimation of the infrastructure performance models described in METHODOLOGY. Strong conclusions should not be drawn from the current study due to unfortunate gaps in the available data.

Data

The data for the study was collected by MnRoad, the road research division of the Minnesota Department of Transportation. MnRoad is at the forefront of using advanced monitoring technologies for condition assessment of pavements. They have an extensive network of sensors that are used to monitor both in-use and experimental pavements that differ in their materials and design characteristics. The sensors collect distress measurements such as the response of pavements to traffic loadings (strain, deflection, pressure, etc.), and several environmental characteristics such as temperature and moisture. MnRoad also uses probes that are equipped with lasers, radar and other technologies to measure pavement roughness, cracking, raveling, and rutting.

The data for the study consist of deflection and pressure measurements from cell 33, a hot-mix asphalt pavement (superpave with PG 58-28) located on a closed-loop test track (see (15) and (16) for additional information). The data are described in the following sections.

Deflection

Pavement surface deflection measurements are the primary means of evaluating a flexible pavement structure. It is measured as a pavement surface's vertical deflected distance as a result of an applied (static or dynamic) load. Deflection measurements are collected by impact/impulse load response, where an impact load device delivers a transient impulse/load to the pavement surface. The induced pavement response (deflection basin) is measured by a series of sensors. The most common type of tool used to induce the load is a Falling Weight Deflectometer (FWD).

The data used in our analysis consist of the monthly average deflection measurements (in thousandths of an inch -- mils) induced by a falling weight deflectometer and collected in the four year period from December 1999 through November of 2003. The data are presented in FIGURE 1 and FIGURE 2. Twelve entries in the data set were reconstructed using the MISDATA interpolation function in MATLAB.

Pressure

Vertical pressure data are used to determine the vertical stress distribution in the base and subgrade layers of a pavement. Strain measurements can be calculated approximately by pressure measurements via modulus of elasticity. Since strain is commonly used to represent pavement strength, pressure data constitute another important indicator of a pavement's structural performance. Pressure measurements in MnRoad are collected by inducing a load generated by a five-axle truck. The pressure measurements (in millivolts) are collected with a dynamic soil
pressure cell sensor. In our analysis, we use the monthly average of the pressure measurements induced by the first axle for the December 1999 through November 2003 period. The data are presented in FIGURE 1 and FIGURE 2. Eight entries were reconstructed using the MISDATA interpolation function in MATLAB.

Estimation Results

ARMA time series models

The data in FIGURE 1 and FIGURE 2 show clear seasonality. The seasonality can be attributed to pavement stiffness, which is high at cool temperatures and low at warm temperatures. Thus, the measurements of deflection and pressure are relatively low during the winter and high during the summer. Due to the seasonality, it is necessary to consider ARMA models of order 12 or higher. To simplify the estimation of the model and the interpretation of the results, we consider AR models instead of ARMA models in the analysis. We set deflection to be the reference measurement, henceforth denoted with subscript 1. We use backward elimination principle to refine the resulting models. The backward elimination principle is to start the estimation with a higher order and eliminate the insignificant parameters to obtain reasonable models.

Several models were considered and the best results were obtained for an AR model of order 14. To illustrate the model selection process, we begin by estimating the general AR(14) model, labeled AR(14)-1. The two most insignificant parameters are set to zero and then AR(14)-2 is estimated (t-statistics with magnitude greater than 2 indicate that the associated parameter is significant at the 95% confidence level). AR(14)-3 and AR(13)-1 are generated in the same way. The parameter estimates and goodness-of-fit measures for all variations are presented in TABLE 1 in the end of the paper. The bold entries are for insignificant parameters. We observe that AR(14)-2 results in the best AIC and FPE. In other words, AR(14)-2 has the best tradeoff of the number of parameters and model fit. Although most of the parameters are insignificant due to the relatively high number of the parameters, AR(14)-2 is the preferred model from the view of goodness-of-fit. Further, from FIGURE 1 the model fits both distress measurements adequately. The residuals of both measurements from AR(14)-2 model are further tested using sample autocorrelation function and the portmanteau test. Sample autocorrelation function shows the residuals are iid with N(0,1/T) since over 95% of the function values are within ±1.96√T. The portmanteau test also shows that the residuals are iid at 95% level of significance. Hence, the preferred AR(14)-2 is statistically satisfactory. As discussed in State-Space Specifications for Time Series Models, one of the disadvantages of ARMA models is that they don't have meaningful components, and thus, no further model interpretation can be made except goodness-of-fit. Note that the measurement error of deflection is much greater than pressure, which can be seen from both the estimated model and the graph. The estimated model has the following form presented in Equations (20) and (21).

\[
x_t = 0.73x_{t-1} - 0.30x_{t-2} + 0.09x_{t-5} - 0.07x_{t-4} - 0.06x_{t-6} + 0.06x_{t-7} + 0.03x_{t-8} + 0.01x_{t-9} - 0.08x_{t-10} + 0.47x_{t-11} + 0.09x_{t-15} + 0.03x_{t-14} + \varepsilon_t, \varepsilon_t \sim N(0,0.03)
\]

(20)

\[
\begin{bmatrix}
    z_{t}^{(1)} \\
    z_{t}^{(2)}
\end{bmatrix} =
\begin{bmatrix}
    1.00 \\
    0.26
\end{bmatrix} x_t +
\begin{bmatrix}
    \xi_{t}^{(1)} \\
    \xi_{t}^{(2)}
\end{bmatrix}, \Sigma_t =
\begin{bmatrix}
    25.00 & -1.89 \\
    -1.89 & 1.08
\end{bmatrix}
\]

(21)

Structural time series models

As described earlier, the data sets show clear seasonality. However, another important component of a structural time series model, the trend, is harder to determine from this data set. To identify the components of the data set, three structural time series models are estimated. The
results are listed in TABLE 2 in the end of the paper. Model BSM-1 incorporates level, trend, seasonality, and irregularity. The estimation shows that the random components, i.e., the variances, of the level, trend, and irregularity are significant. In addition to the level, trend, seasonality, and irregularity the second model, BSM-2, incorporates the one-step lagged, dependent variable, which turns out not to be significant. Also, BSM-2 is worse than BSM-1. The third model, BSM-3, incorporates both the one and two-step lagged, dependent variables. Comparing BSM-3 and the other two models in TABLE 2, virtually all the variances of the random error terms are lowered after adding the lagged, dependent variables, i.e., the model captures the transition of the system better and the model itself becomes “less stochastic”. In other words, the lagged, dependent variables in the model explain a larger part of the variance in the infrastructure performance data. Note that the goodness-of-fit measures are better than those for the other two models. Thus, BSM-3 is preferred among BSM models. Moreover, the fact that the two-step lagged, dependent variable is highly significant indicates that the Markovian assumption does not hold. The fact that the one-step lagged, dependent variable is not significant (see TABLE 2 can be explained by the inclusion of the “trend” variable in the model.

FIGURE 2 also shows that the model fits the data adequately. The beauty of the structural time series models is the meaningful components of the series, which is shown clearly in FIGURE 3. In the figure, the level, trend, and seasonality are clearly decomposed. The seasonality of the state matches of seasonal change of the deflection measurements reasonably. The negligible trend matches the fact that both the measurements are not increasing or decreasing except for the seasonal change. In other words, the section didn't deteriorate in the period of experiment. The level of the state is higher than the deflection measurements because the effect of the lagged dependent variables in the system equation. Moreover, the residuals of both measurements from BSM-3 model are further tested using sample autocorrelation function and the portmanteau test. Both tests show that the residuals are iid, which indicates that the preferred BSM-3 model is statistically satisfactory. The preferred model has the following form shown in Equations (22) through (26).

\[
\begin{align*}
x_t = 0.13x_{t-1} - 0.63x_{t-2} + \mu_t + \gamma_t + \epsilon_t, \quad \epsilon_t \sim N(0, 0.09) \\
\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_{t-1}, \quad \eta_t \sim N(0, 0.26) \\
\beta_t = \beta_{t-1} + \xi_{t-1}, \quad \xi_t \sim N(0, 0.01) \\
\gamma_t + \gamma_{t-1} + \gamma_{t-2} + \cdots + \gamma_{t-r} = \omega_t, \quad \omega_t \sim N(0, 0.0005) \\
\begin{bmatrix}
\xi_{t-1}^{(1)} \\
\xi_{t-1}^{(2)}
\end{bmatrix} =
\begin{bmatrix}
1.00 \\
0.93
\end{bmatrix} x_t +
\begin{bmatrix}
\xi_{t-1}^{(1)} \\
\xi_{t-1}^{(2)}
\end{bmatrix}, \quad \Sigma \xi =
\begin{bmatrix}
15.52 & -2.54 \\
-2.54 & 2.20
\end{bmatrix}
\end{align*}
\]

CONCLUSIONS

In this paper, we propose state-space specifications of two classes of time series models as a framework to develop and estimate performance models for transportation infrastructure facilities. The framework is consistent with the latent performance modeling approach of (1), which means that it rigorously accounts for uncertainty in forecasting condition, both in the deterioration and in the data-collection process, when data are gathered by multiple technologies.

The performance models described herein fit the optimization model presented by Durango-Cohen and Tadepalli (5) to support investment decisions in M&R of transportation infrastructure. This, in turn, means that they constitute alternatives to the use of Markovian transition probabilities. In LITERATURE REVIEW of the paper, we argue that the use of time series may be attractive to develop and estimate infrastructure performance models due to characteristics that make the MDP framework statistically unattractive to forecast infrastructure...
condition. This is in addition to the computational shortcomings of the MDP framework that are described in detail in Durango-Cohen (4).

Finally, we present an empirical study where we validate the proposed methodology to forecast infrastructure condition when multiple technologies are used simultaneously to gather condition data. Specifically, we estimate and compare autoregressive and structural time series models to simultaneously forecast deflection and pressure measurements. The data for the empirical study, provided by MnRoad, was gathered using sensors implanted in an asphalt pavement. In addition to verifying that the proposed models provide adequate and reasonable forecasts, the results of the empirical study show that the history of the deterioration process is statistically significant. This result is consistent with other recent studies, and raises questions about the adequacy of the MDP approach.

ACKNOWLEDGMENTS

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FIGURE 3 Decomposition of State for BSM-3
TABLE 1 Estimation results for ARMA models with two distress measurements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>AR(14)-1</th>
<th>AR(14)-2</th>
<th>AR(14)-3</th>
<th>AR(15)-1</th>
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<td></td>
<td>Estimate</td>
<td>t-stat</td>
<td>Estimate</td>
<td>t-stat</td>
</tr>
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<td>$\phi_1$</td>
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<td>1.63</td>
<td>0.73</td>
<td>2.05</td>
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<td>$\phi_2$</td>
<td>-0.35</td>
<td>-0.51</td>
<td>-0.3</td>
<td>-0.56</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>0.03</td>
<td>0.07</td>
<td>0.09</td>
<td>0.20</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>-0.05</td>
<td>-0.19</td>
<td>-0.07</td>
<td>-0.32</td>
</tr>
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<td>$\phi_5$</td>
<td>-0.01</td>
<td>-0.04</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$\phi_6$</td>
<td>-0.03</td>
<td>-0.13</td>
<td>-0.06</td>
<td>-0.34</td>
</tr>
<tr>
<td>$\phi_7$</td>
<td>0.04</td>
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<td>0.22</td>
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<td>$\phi_8$</td>
<td>0.02</td>
<td>0.10</td>
<td>0.03</td>
<td>0.08</td>
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<td>$\phi_9$</td>
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<td>0.04</td>
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<td>$\phi_{10}$</td>
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<td>-0.06</td>
<td>-0.08</td>
<td>-0.45</td>
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<td>$\phi_{11}$</td>
<td>0.39</td>
<td>1.93</td>
<td>0.47</td>
<td>3.40</td>
</tr>
<tr>
<td>$\phi_{12}$</td>
<td>0.01</td>
<td>0.04</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_{13}$</td>
<td>0.07</td>
<td>0.26</td>
<td>0.09</td>
<td>0.37</td>
</tr>
<tr>
<td>$\phi_{14}$</td>
<td>0.06</td>
<td>0.21</td>
<td>0.03</td>
<td>0.21</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.24</td>
<td>3.04</td>
<td>0.26</td>
<td>3.50</td>
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<tr>
<td>$\sigma_2^2$</td>
<td>0.05</td>
<td>0.98</td>
<td>0.03</td>
<td>0.65</td>
</tr>
<tr>
<td>$\sigma_5^2$</td>
<td>23.86</td>
<td>25.99</td>
<td>29.57</td>
<td>28.07</td>
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<tr>
<td>$\sigma_6^2$</td>
<td>4.14</td>
<td>3.49</td>
<td>3.80</td>
<td>4.30</td>
</tr>
</tbody>
</table>

where:

$\sigma_2^2$: Variance of the system equation.

$\sigma_5^2$: Variance of the deflection measurement.

$\sigma_6^2$: Variance of the pressure measurement.
## TABLE 2 Estimation results for basic structural model (BSM) with two-step lagged dependent variable

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BSM-1</th>
<th>BSM-2</th>
<th>BSM-3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>t-stat</td>
<td>Estimate</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>-</td>
<td>-</td>
<td>0.07</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>0.31</td>
<td>13.46</td>
<td>0.28</td>
</tr>
<tr>
<td>( \sigma_e^2 )</td>
<td>0.14</td>
<td>2.13</td>
<td>0.20</td>
</tr>
<tr>
<td>( \sigma_e^2 )</td>
<td>0.31</td>
<td>7.2</td>
<td>0.71</td>
</tr>
<tr>
<td>( \sigma_c^2 )</td>
<td>0.12</td>
<td>10.46</td>
<td>0.01</td>
</tr>
<tr>
<td>( \sigma_o^2 )</td>
<td>0.0003</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>( \sigma_e^2 )</td>
<td>25.81</td>
<td>19.51</td>
<td>15.52</td>
</tr>
<tr>
<td>( \sigma_e^2 )</td>
<td>2.12</td>
<td>3.47</td>
<td>2.20</td>
</tr>
<tr>
<td>FPE</td>
<td>75.21</td>
<td>81.16</td>
<td>48.34</td>
</tr>
<tr>
<td>AIC</td>
<td>4.27</td>
<td>4.34</td>
<td>3.62</td>
</tr>
</tbody>
</table>

where:

\( \sigma_e^2 \): Variance component of the system equation.

\( \sigma_o^2 \): Variance component of the level equation.

\( \sigma_c^2 \): Variance component of the trend equation.

\( \sigma_e^2 \): Variance component of the seasonality equation.

\( \sigma_{\epsilon_i}^2 \): Variance of the deflection measurement.

\( \sigma_{\epsilon_i}^2 \): Variance of the pressure measurement.
FIGURE 1 Model vs. Data for AR(14)-2

- Observed Deflection
- Predicted Deflection
- Observed Pressure
- Predicted Pressure

[Graph showing observed and predicted deflection and pressure values over months from July 1999 to December 2003.]
FIGURE 2 Model vs. Data for BSM-3

- Observed Deflection
- Predicted Deflection
- Observed Pressure
- Predicted Pressure

Month

- Jul-99
- Jan-00
- Jul-00
- Jan-01
- Jul-01
- Jan-02
- Jul-02
- Dec-02
- Jun-03
- Dec-03

mils (deflection)
millivolts (pressure)
FIGURE 3 Decomposition of State for BSM-3

Chu and Durango-Cohen